STOCK PORTFOLIO OPTIMIZATION ON THE CAPITAL MARKET

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Summary

A this paper aims to examine various models for optimizing investment in shares on the capital market in Romania. The research is of interest to both financial institutions such as investment funds, pension funds or banks, as well as to individual investors. L ucrare to be divided into three sections. The first chapter provides a brief theoretical description of the portfolio theory regarding capital market optimization and the presentation of relevant specialty literature. The second chapter represents the methodology of the case study and the description of the data set, more precisely the constitution of the portfolio of actions. The last chapter represents the applicative part of this work. The results and analysis of portfolio optimization methods will be presented here.

Keywords: optimization, diversification, stock portofolio, value-at-risk

JEL Clasification: G11, G17

Introduction

In any society the capital market plays an important role in economic development. Low financial intermediation can keep a state in economic underdevelopment by the fact that the economies of the population are not efficiently allocated. Developing the capital market can be an important step in enhancing investments and sustainable economic growth. Unfortunately, the increased risks related to investments in the Romanian capital market can discourge investors. This is why it is necessary for any type of investor to know how to get certain returns with minimum risks. Therefore, this paper aims to analyze various models of optimization of investments in shares on the Romanian capital market. The research is of interest to both financial institutions such as investment funds, pension funds or banks, as well as to individual investors.

The optimal portfolio concept emerged with the modern theory of the portfolio. Among other things, this theory implies that investors focus their efforts on minimising risk or maximising profitability on the risk unit.According to this theory, investors will act rationally within these parameters and will always make decisions in order to maximize profitability for a certain acceptable level of risk. Harry Markowitz was the first to present the optimal portfolio idea in 1952. This model shows that different portfolios may have different levels of risk and profitability. Thus, individual investors should determine how much they are willing to take risks and then allocate or diversify their portfolios based on the results of the respective decision.

However, in practice, the model proposed by Markowitz is not very used because it requires a large amount of information and is based on a series of assumptions that can hardly be fulfilled, especially in the case of less developed markets. Therefore, several alternative models have been developed that satisfy the profitability-risk balance. In the specialized literature, various articles have appeared that offer a research on the optimization models or a comparison of them, considering that the results regarding the optimization of the portfolio differ from one model to another. It can be said that choosing an optimization model is based on the risk aversion of each investor. However, we aim to see to what extent these models can be applied on the local market, if they offer credible results and if there is a possibility that one model will perform better than the others.

1. Review of the specialized literature

Over time, optimization models have been analyzed by both capital market researchers and practitioners. The specialized literature is particularly rich both on the international market and on the Romanian market. Many specialized articles have tried to identify which optimization models perform in certain markets and under certain conditions. An important article in this regard was made by Koegelenberg (2012) who uses a number of models to optimize pension funds. This study uses 7 optimization models:

- Equal weight portfolio;
- Equal weight portfolio of the contribution to the risk ;
- The traditional Markowitz mean ~ variance;
- Re-sampling the optimization of the mean ~ variance ;
- Optimization by using Extreme Value Distribution;
- Optimization by using Value at Risk;
- Optimization by nonparametric methods .

Finally, the optimal portfolios of each model are compared to see if there is an optimization model that generates the best portfolio. To compare the different models you need an optimal portfolio resulting from each model used. This is achieved using the risk-return rate, the risk being the standard deviation for the models where a frontier of efficient portfolios results. If an efficient frontier is not used for the optimization process, the risk is calculated using VaR and CVaR. The portfolio that has the best risk-return rate for each optimization process will then be used to compare different optimization methods. The portfolio that offers the highest probability of exceeding 4% yield is the best portfolio and therefore the best optimization method. The final results indicated that the best method is to re-sample the optimization of the average ratio ~ variance, using Value-at-Risk in optimizing the portfolio with the worst results.

Another important article is that of Miskolczi (2016). It analyzes the difference between the mean-variance method and the method of using Value-at-Risk by using daily data on shares in Hungary and tries to answer the question whether the decision on portfolio composition differs in the two cases. The analysis shows that there is a difference and that the investment decision differs depending on the risk measure. Risk value (VaR) is the most widely used measure of risk in recent years. Despite the widespread use of Value at Risk, there are certain fundamental problems with it, because it does not satisfy the property of subadaptivity and ignores the gravity of the losses. Moreover, the convexity makes VaR impossible to use in optimization problems. Thus, *expected shortfall* (ES) has developed as a coherent risk measure. The author uses this measure to optimize the portfolio. Expected shortfall, is also called Conditional value at risk (CVaR). Compared to VaR, ES is more sensitive to the behavior of the extremes in the "queue" of the profit distribution function.

Kathleen Ferguson and Brian Rom (1993) presented in the paper " *Post-modern portfolio theory comes of age* " a new approach in portfolio optimization based on the general rules allowed by post-modern portfolio theory and the limitations of modern time :

• The variance of the portfolio's profitability is the correct measure of the investment risk;
• The returns on financial assets follow a normal distribution.

The paper presents a comparison between the modern theory using the medium-variance method and the post-modern theory, using the semi-variance in order to optimize a portfolio that includes high capitalization companies, low capitalization companies, foreign companies, bonds and cash. The two conclude that optimization by the mean-variance method can produce illogical and counter-intuitive results and demonstrate how post-modern theory can remedy these problems. For example, by applying the average-variance model, the portfolio with the absolute minimum variance is composed almost entirely of cash, while the half-variance model also indicates the inclusion in the portfolio of high-capitalized shares and bonds. Another difference identified refers to the fact that the shares of the high capitalization companies with a higher skewness are more "attractive" in the optimization using the half-variance, where the skewness is recognized, whereas in the average-variance model it is not recognized. By providing a more precise and robust framework for building optimal portfolios, the post-modern portfolio theory has made the necessary improvements to Markowitz and Sharpe's fundamental theory.

There are also internally a number of articles and researches in portfolio optimization. A reference article on this topic is "Optimizing the allocation of assets of privately administered pension funds in Romania" (Stancu, Badea and Darmaz-Guzun, 2018), which offers an analysis on the optimization of the portfolio on the local market. The study reveals the optimum weights for allocating pension funds in certain categories of assets (shares, bonds, funds, government securities and bank deposits) by using three optimization models: the simple capitalized portfolio, the standard deviation minimization (Markowitz) and the value minimization at risk (CVaR). The results of the study show that the balanced portfolio is recommended to private pension managers but also to investors in the capital market who take a low risk. Within the mean-variance model, pension fund managers aim to optimize the portfolio by minimizing the standard deviation. As a result, they can opt for the optimal allocations in the five categories of placements at a desired level of profitability expected depending on the degree of risk assumed. In the medium-value-risk model, investors are concerned about the maximum cumulative loss that can be recorded with a probability of 10%. As a result, they can opt for a discounted return that minimizes this loss, depending on the investors' aversion to risk. Another article on the local market (Andreescu 2017) demonstrates the positive impact of the optimization of the pension funds on the development of the Romanian economy.

Following the research of specialized articles and books in the field of portfolio optimization, we selected a series of models that will be applied to the Romanian capital market, a frontier market that is trying to go beyond this stage and become an emerging market. This can be achieved by increasing investor confidence in this market.

Demonstrating the usefulness of optimization in this market can contribute to increasing investor confidence.

2. Methodology and data series

2.1. Methodology

The purpose of this work, as it is shown in the title, is to optimize a portfolio of shares on the local capital market. Given the multitude of optimization methods of the financial instrument portfolios used and analyzed in the specialized articles presented in the previous chapter, we have proposed an analysis on these models on the Romanian stock market.

The Markowitz model, mean-variance

The Markowitz model involves portfolio diversification by obtaining a series of combinations of securities and leads to the identification of efficient portfolios so that the required profitability is achieved by assuming a minimum risk. In order to generate portfolios efficiently , we will calculate the structure of a portfolio that will provide the required average profitability with minimal risk. Thus, the weights of the various securities in the portfolio with the expected profitability on the efficient Markowitz frontier become known and the risk of the portfolio can be found. Markowitz's efficient frontier starts with **the minimum risk portfolio (PVMA)** expected by investors with the highest risk aversion, which separates the set of possible portfolios into two sub-segments:

- **efficient portfolios,** located above the PVMA and which associate with each increase in portfolio risk an (uneven) increase in expected profitability;
- **inefficient portfolios,** located under the PVMA and that associate with each increase of risk of bone portfolio a fall in expected profitability.

Sortino model, mean-semivariance

The postmodern theory of the portfolio develops new models that can better reflect the evolutions of the financial markets because the events that make up the evolution of the returns are asymmetrical . The theory was introduced by Sortino in 1980 introducing a new measure of risk: the half-variance as a possibility of a lower return than the average return expected by the investor (Minimum Acceptable Return - MAR).

Given that an investor is more concerned with avoiding loss than seeking a return, or in other words, a return above the expected average is normal and represents a premium for the investment , we will consider the following:

$$MAR = R_f \quad (1)$$

$$R_i - R_f < 0 => R_i - R_f \quad (2)$$

Thus, we compose new series of data consisting only of the returns below the risk-free rate on which we will apply the minimum problem according to the algorithm used and to the Markowitz model. Therefore, within the optimization model, a new variance-covariance matrix is composed , and the expected return is replaced by the difference between the expected return and the risk-free rate. Thus, the weights of the various titles in the portfolio with the profitability I would expect to find on the efficient Sortino frontier become known .

Sharpe model (Capital Market Line, CML)

The Sharpe model simplifies calculations by reporting stock returns to the market index. We will assume that on the market there is a risk-free asset $\boldsymbol{R}_f$ whose yield is certain and the risk is zero, which we will include in the portfolio. We considered the risk-free rate as the interest rate of the state bonds maturing at 6 months 3.71%. Thus, by combining risk-free assets with risky assets, the risk-return relationship becomes the fundamental right of the capital market (*CML*).

In this model we will consider what are the two cases in which investors have Markowitz type behavior according to MPT and Sortino type according to PMTP:

- **Sharpe-Markowitz model (CML + MV)** - Efficient portfolios are found on the tangent taken from point Rf at the Markowitz frontier. These portfolios dominate the Markowitz frontier because at the same risk, the profitability obtained is higher .

- **Sharpe-Sortino model (CML + MSV)** - The expected return on the portfolio is calculated as the difference between the expected return and the risk-free rate. Also, the efficient portfolios can be found on the tangent taken from the Rf point at the Sortino frontier.

Method of using conditional value at risk (Mean-Conditional VaR)

The M-CVaR model is a preferred method for protecting against the risk of loss of a portfolio. This model minimizes conditional value at risk for expected return. Conditional Value at Risk is a measure of risk assessment that quantifies the amount of risk.

CVaR is calculated as the average of the "extreme" losses in the tail of the distribution of returns , greater than Value-at-Risk (VaR). Value at risk shows what is the maximum loss of a portfolio over a given time horizon and a predetermined level of confidence. In this case we chose the historical method for calculating VaR. This method reorganizes the historical returns of a portfolio by placing them in ascending order. Then, using the PERCENT formula in Excel will determine the maximum value that can be lost on a given probability day. To determine the structure of a portfolio that minimizes the risk-adjusted value for a discounted profitability, we will use the SOLVER subroutine in Excel.

Probability of recording portfolio performance

Portfolios that have the highest risk-adjusted yield for each optimization process will then be used to compare optimization methods. The best portfolio will be achieved by using a procedure that will determine the probability of recording performance, more precisely the probability of a portfolio reaching a return threshold. The portfolio that provides the highest probability of performance is the best portfolio and therefore the best optimization method. Assuming that the yields of an asset are normally distributed, they can be expressed according to standard normal distribution. We can associate the distribution of profitability with a standard normal distribution, which has an average 0 and standard deviation 1. Therefore, the retention can be expressed as follows:

$$r = \mu_r + Z\sigma_r \quad (3)$$

$$Z = (r - \mu_r)/\sigma_r \quad (4)$$

The probability will be calculated using the NORM function. S. DIST in Excel.

2.2. Data series

Before carrying out an analysis of the allocation decision, the selection of the titles to be included in the portfolio is very important. In view of the low liquidity, we will direct ourselves to the companies that are part of the BET-Plus index of the Bucharest Stock Exchange. The index currently consists of 37 companies, which is revised quarterly. From the start we exclude 5 companies, they are listed in the last 3 years. Given that models are based on a number of assumptions, it is necessary to apply additional selection criteria on the profitability and liquidity of actions. For the analysis of our research we have taken over the closing prices from 04.04.2016-04.04.2019 (752 observations) of the selected actions taken from the www.investing.com. We also considered the risk-free rate as the interest rate of state tilages at 6 months: 3.17% on 05.04.2019. The data needed to select the actions included in the portfolio have been retrieved from the www.bvb.ro.

The first step in building a portfolio is the choice of titles. In order to have a relevant analysis of the profitability and risk of securities, it is necessary that they meet a number of liquidity criteria, transaction volumes and correlation between the titles included in the portfolio. The selection criteria applied to the titles are as follows:

- Annual average profitability > Rf
- Stock capitalisation > 25 mil EUR – Premium category
  > 5 mil EUR – Standard category
- Average number of transactions per day > 25 – Premium category
  > 5 – Standard category
- Average transaction value per day > 40.000 RON – Premium category
  > 8.000 RON – Standard category

Table No. 1 – Criteria for the selection of titles

Company	Category	Rentab.	Capitalization (EUR)	No. Tranz.	Val tranz.(RON)
TURBOMECANICA SA	Standard	51.24%	22,543,095	27.54	47,242
ELECTROARGES SA	Standard	21.67%	18,349,074	8.33	19,742
NUCLEARELECTRICA SA	Premium	20.50%	657,257,816	106.32	795,320
ALRO SA	Premium	20.41%	339,422,809	55.40	165,891
AEROSTAR SA	Standard	20.12%	157,000,274	5.50	15,283
OMV PETROM SA	Premium	14.14%	4,374,108,437	137.77	4,018,190

IAR SA BRASOV	Standard	13.58%	31,381,336	7.68	18,673
OIL TERMINAL SA	Standard	10.68%	16,299,125	8.86	10,571
BRD - GSG SA	Premium	8.63%	1,961,987,609	151.86	5,227,722
SNTGN TRANSGAZ SA	Premium	6.99%	879,458,532	91.76	1,745,884
SNGN ROMGAZ SA	Premium	6.85%	2,708,645,406	153.59	3,820,388
FONDUL PROPRIETATEA	Premium	6.29%	1,796,414,918	57.11	3,789,938
IMPACT DEVELOPER SA	Premium	5.01%	56,946,101	27.75	104,946
COMPA S A	Standard	4.63%	34,071,364	16.71	60,249
VRANCART SA	Standard	4.56%	35,166,589	6.18	8,262
BANCA TRANSILVANIA SA	Premium	4.07%	2,203,599,639	383.68	11,781,001

Source: Data retrieved from www.bvb.ro and processed in Excel

Following the application of selection criteria resulted in a number of 16 companies from the 32 included in the BET-Plus index and listed more than 3 years ago. The selection of titles also offers a diversification on economic sectors. Thus, 7 companies in the industrial-technological sector, 5 in the energy sector, 3 in the financial sector and 1 in the construction sector were selected. Another important step in the construction of the portfolio is to identify those heavily related titles and remove one of them. It follows from the correlation matrix that the shares are not heavily correlated, with the only correlation coefficient greater than 0.5 between the BRD and TLV actions, both of which are companies in the banking sector. We will remove it from the TLV portfolio, with a lower annual return.

3. Analysis of empirical results

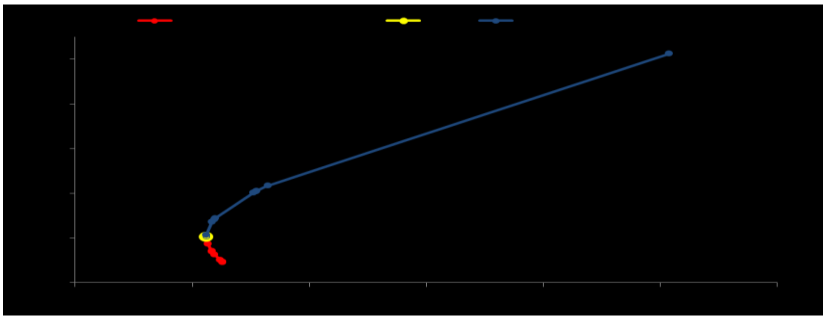
3.1. Optimizing the Stock Portfolio

In order to optimize the portfolio we have chosen 4 models to identify what differences exist between them and how we can identify the best model. Markowitz, Sortino and C-VaR models optimize the portfolio in the assumption that in the market the Invesitors do not have access to the risk-free asset, and the Sharpe model presupcovers its inclusion in the portfolio.

3.1.1. Markowitz model

For portfolio optimization using the average model-Markowitz variance We simulated the expected profitability of each title included in the portfolio resulting in 15 portfolios. Effective portfolios start from the portfolio with absolute minimum variance (PVMA) that has the highest aversion at risk.

Effective portfolios start from expected profitability of 10.3% and as they increase, the voltatility of the portfolio is higher. For lower expected profitability, the share of less volatile titles such as FP is higher. Also, for high profitability, the model indicates higher weights for more volatile actions. Portfolios with expected profitability between around 13% and 15% are all portfolios that have in their composition shares with positive weights. For other expected profitability, portfolios have negative weights, with investors having to turn to short-sell operations, this type of operation is also available on the regulated market of BVB. Below we illustrated graphic border of portfolios optimized by Markowitz model:



Graph No. 1 – Markowitz Frontier

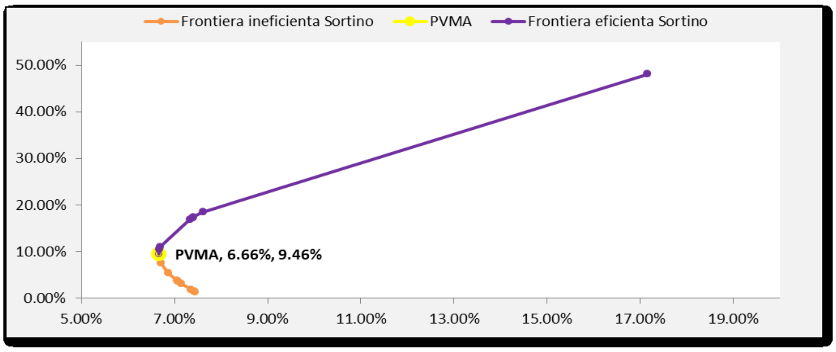
Source: Excel's own processing

From the graph it is noted that investing in portfolios on the inefficient frontier can achieve the same profitability as investing in portfolios on the efficient frontier but with greater risk taking.

3.1.2. Sortino model

According to the Sortino model we considered the measurement of risk by the semi-annuation calculated as the variance of the minimum profitability acceptable to which we will consider the risk-free asset rate. To optimize the portfolio using the Sortino model we simulated the expected profitability of each title included in the portfolio resulting in 15 portfolios of which 7 are located over the PVMA being effective portfolios.

Effective portfolios start from expected profitability of 9.46% (over Rf) and as they increase, the risk of portfolios increases. Portfolios with expected profitability between around 9% and 11% are portfolios that do not have shares with negative weights, and short-sell operations are not required. Graph 2 illustrates the frontier of portfolios optimized by the Sortino model:



Graph No. 2 – Sortino Frontier

Source: Excel's own processing

3.1.3. Sharpe-CML Model

The model implies the introduction of the risk-free asset in the construction of optimal portfolio. Thus, the risk-profitability relationship becomes the fundamental right of the capital market (CML), which represents the frontier of optimal portfolios including the risk-free rate. Considering that investors have a Markowitz behavior we simulated expected profitability of the actions resulting in 15 optimal portfolios located on the right of the CML.

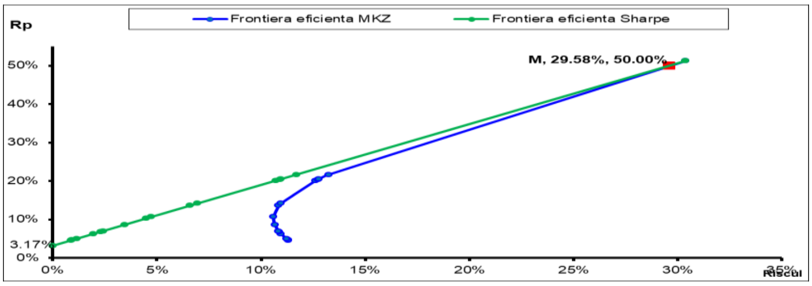
The weight of the risk-free asset is very high up close to the expected profitability of around 50% which proves the high risk of securities included in the portfolio. The risk is also lower when we include the asset without risk and increases with the expected profitability. For example, if an investor wants a 10.26% return, it must allocate 84.87% in the risk-free asset taking a risk of 4.48%. From a certain level of profitability expected the risk-free asset has negative weightîn portofoliu. The 0% risk-free asset share is the market portfolio consisting only of risk assets. The structure of the market portfolio is as follows:

Table No. 2 – Structure of the market Walltab

Market portofolio	E(Rp)	Rf	VNC	CMP	IMP	FP	SNG	TGN	BRD
	50.00%	0.00%	-13.8%	-7.3%	-5.6%	-20.6%	-18.8%	-12.9%	-0.4%
	OIL	IARV	SNP	ARS	ALR	SNN	ELGS	TBM	σ_{an}
	4.9%	-6.9%	9.6%	33.4%	8.9%	46.8%	22.6%	60.1%	29.58%

Source: Excel's own processing

Graph no. 3 illustrates the frontier of the CML portfolios at the intersection with the Makowitz frontier at point M:

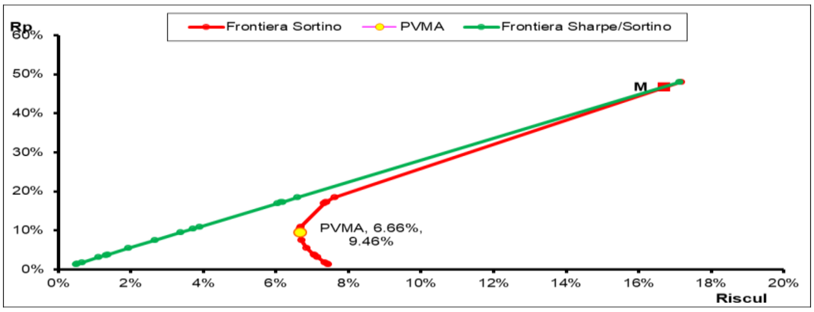


Graph No. 3 – Sharpe-Markowitz frontier

Source: Excel's own processing

Effective portfolios are found right CML which is the tangent taken from the R_f point at the Markowitz frontier. This frontier dominates the Markowitz frontier because at the same risks, the profitability achieved is higher.

If we believe that investors have a Sortino type of behaviour, the portfolios on the CML have a lower requested return by taking into account the risk-free asset rate. We note that for the expected cost-effectiveness of the previous case, from which the risk-free asset rate is deducted, the same weights are obtained for the portfolio headings, the risk assumed being the semivariance. Also, the market portfolio with a expected return of 46.83% and a Sortino risk of 16.7% consists only of risk assets having the same structure as in the previous case. In this case the CML right starts from point 0 and is tangent to the Sortino frontier at Point M. Graph no. 4 illustrates the two frontiers:



Graph No. 4 – Sharpe-Sortino frontier

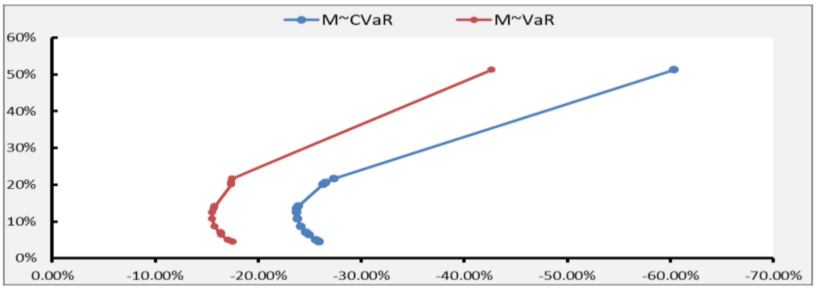
Source: Excel's own processing

It is noted that the right of the CML departs from Point 0, investor expectations being in this case above the risk-free asset rate. The right is tangent to the Sortino frontier at Point M having the coordinates 46.83% on the expected profitability axis and 16.7% on the risk axis calculated as a semivariance.

3.1.4. Method of using Conditional-Value at Risk

In this model, optimization can be achieved by maximizing profitability for a given level of C-VaR or by minimizing it for a certain level of expected profitability. In this paper we optimized the portfolios by minimizing C-VaR for the profitability levels of individual actions using the Solver subroutine in Excel. The confidence interval used in VaR calculation is 95%.

The optimum combination with the lowest annualised loss (CVaR = – 23.72%) Corresponds to an estimated cost of 12.48%. Clearly increasing the expected level of profitability in this point leads to increased risk-conditional value. For a level of 51.27% of profitability, the value at risk is 42.67%, while the average of losses exceeding this level is 60.4%. Below we have illustrated the two frontiers: expected profitability – value at risk and expected profitability – conditional value at risk.



. **Graph nr. 5 – M-VaR și M-CvaR frontier**

Source: Excel's own processing

From the graph it is noted that investing in portfolios on the inefficient frontier can achieve the same profitability as in the case of investing in portfolios on the efficient frontier but with risk taking. Investors can orient themselves to the optimal portfolios leading to the expected profitability in relation to the degree of risk assumed (conditional risk value). Choosing this portfolio Optimization model (min CVaR) leads to different compositions compared to the other optimization models used for the same profitability.

3.2. Performance of Optimal portfolios

We have seen that the use of optimization models obviously lead to different weights for the same expected profitability. This way, the question arises whether a better performing optimization model exists. Choosing the optimization model depends on how an investor charges the risk and the optimum portfolio of risk aversion. However, within each model, on the frontier of effective portfolios we will try to identify the portfolio with maximum efficiency. Subsequently, we will calculate the probability of recording performance for the highest efficiency portfolios on each frontier and for the lowest risk portfolios, thus identifying the model with the best result.

In general, the market-based invesitors have the objective of achieving profitability higher than the market. We will therefore consider that the objective of an investor is to exceed the average annual profitability of the BET-Plus Market Representative index (6.76%). We will also consider the return thresholds with values between 20%-50% to highlight the performance of the portfolio where high profitability is expected.

Table No. 3-Portfolio performance with maximum efficiency

		Markowitz Max efficiency	Sortino Max efficiency	C-VaR Max efficiency
μ an		43.09%	40.58%	43.46%
σ an		25.16%	24.21%	26.85%
Prob. μ>E(Rp)	6.76%	92.56%	91.88%	91.42%
	20.00%	82.06%	80.24%	80.89%
	30.00%	69.85%	66.89%	69.19%
	40.00%	54.88%	50.95%	55.13%
	50.00%	39.17%	34.85%	40.38%

Source: Excel's own processing

It is noted that for low return thresholds, the likelihood of exceeding the target is higher. For example, the probability that the portfolio with maximum effectiveness exceeds 6.76% profitability is 92.56%. Instead, the portfolio obtained by minimizing CVaR has higher probabilities than the other portfolios exceed the profitability of more than 40%.

In the case of riscophobic investors it is more important which model performs better in the event of absolute minimization of risk. Thus, we have calculated the probability of recording performance for portfolios with absolute minimum risk for each model:

Table No. 4-Minimum Risk portfolio performance

		Markowitz PVMA	Sortino PVMA	C-VaR PVMA
μ an		10.26%	12.63%	12.48%
σ an		10.60%	11.05%	11.20%
Prob.	6.76%	62.94%	70.25%	69.53%

μ>E(Rp)	**20.00%**	17.90%	25.24%	25.12%
	30.00%	3.13%	5.80%	5.90%
	40.00%	0.25%	0.66%	0.70%
	50.00%	0.009%	0.036%	0.041%

Source: Excel's own processing

It is noted that for low profitability thresholds the Sortino model registers higher probability that profitability exceeds the target. For example, the probability that the maximum effective portfolio exceeds the profitability of 6.76% is 70.25%. On the other hand, the portfolio obtained by minimizing CVaR has higher probability than the other portfolios exceed the profitability of more than 30%, which is irrelevant for portfolios pursuing the absolute minimum risk.

In conclusion, analyzing optimized portfolios, models lead to different structures for the same expected profitability, the choice of the investor is based on its risk aversion. For balanced investors on their risk aversion and wishing to achieve high efficiency profitability thresholds, the traditional Markowitz model is recommended, and for investors aiming at minimising risk, the The postmodern theory model of the Sortino portfolio which aims at minimizing the lower half-variance of the accepted minimum profitability. Also, the minimization model of the conditional value at risk provides close results to the models, being recommended for the investments of riscophiles pursuing high profitability.

3.3. Impact of optimising on Value at Risk

In this chapter of the research we will try to highlight the impact of optimising the value at risk calculated as the maximum loss of the portfolio. We will therefore consider the situation of the equipated portfolio compared to the portfolio optimized by the traditional Markowitz model used in the preceding subchapter. The average annual profitability for the equip portfolio is 14.35%. Therefore, through the Markowitz model we minimize the variance for the expected profitability 14.35%. For the two portfolios we will calculate value at risk through the analytical method using the GARCH model for estimating volatility.

3.3.1. Features of portfolio profitability

Before using the models to estimate the variation required in the VaR calculation, we will analyse the daily profitability series of both the equip and the optimum portfolio of stationary, the normality of distribution and volatility.

For testing the series stationary I used the ADF test. We have tested the null hypothesis of non-state. The probability associated with the ADF test is below the relevance level of 1%, so we can reject the null hypothesis, the series is stationary, the average and the variance of the series being constant over time, both for the team and optimum portfolio. For testing the normality of the series distribution I used the Jarque-Bera test. The results obtained indicate that the distribution of profitability is not normally distributed, but Leptokurtotic (Kurtosis > 3). Skewness is negative and indicates that the distribution is tilted to the right with more extreme values to the left. This tells us that negative news has a greater impact on volatility than positive news. The Non-normality of distribution suggests that using normal distribution cuantiles in VaR calculations will underestimate the risk. In the case of the optimal portfolio Markowitz we have asymmetric leptokurtotic distribution with the probability of higher negative values. However, the Kurtosis indicator is smaller than the gear portfolio, and Skewness closer to 0, the distribution was more than normal, and the estimated risk will be less underestimated.

Taking into account the series ' stationarity, the phenomenon of volatility clustering, but also the leptokurtotic distribution of yields, the VaR measures calculated in the normal hypothesis will underestimate the risk more for the equipated portfolio. This suggests the use of GARCH conditional variance modelers for the calculation of VaR.

3.3.2. Estimation of portfolios volatility through the GARCH model

For the equal weight portfolio we estimated GARCH models considering p and Q comprised between 1 and 3. For each model I considered the following distributions of errors: normal, T-Student and GED (Generalized Error Distribution). Only models with statistically significant coefficients will be selected, and with the help of Akaike and Schwarz information criteria (priority, as it penalises more the loss of degrees of freedom when adding the Paramates) will choose the best Model.

Table No. 5-GARCH models for equal weight portfolio

Model	Error distribution	Akaike info criterion	Schwarz info criterion
GARCH(1,1)	Normal	-7.026	-7.001
GARCH(1,2)	Normal	-7.029	-6.999
GARCH(2,1)	Normal	-7.034	-7.003
GARCH(3,2)	Normal	-7.035	-6.992
GARCH(1,1)	**t-Student**	**-7.205**	**-7.174**
GARCH(2,3)	t-Student	-7.210	-7.161
GARCH(1,1)	GED	-7.189	-7.158

Source: Eviews's own processing

By selecting only models with statistically significant coefficients (prob. coef. < 0.05), following the minimisation of informational criers, to estimate the variance of the equip portfolio, we chose the GARCH Model (1.1) with the distribution of T-Student errors. On this model we applied the test on the correlation of squares, the heteroskedasticity test of the residence and the normality test of the residence for determining whether the estimated variance is statistically significant with a level of Confidence of 95%.

We also estimated GARCH models considering p and Q comprised between 1 and 3 for the Markowitz optimum portfolio. For each model we considered the three distributions of errors, the model being selected using informational criteria.

Tabel no. 6 - GARCH models for Markowitz Optimum portfolio

Model	Error distribution	Akaike info criterion	Schwarz info criterion
GARCH(1,1)	Normal	-7.200	-7.176
GARCH(1,2)	Normal	-7.208	-7.177
GARCH(2,1)	Normal	-7.209	-7.179
GARCH(1,1)	t-Student	-7.372	-7.342
GARCH(1,1)	GED	-7.362	-7.331
GARCH(1,3)	GED	-7.363	-7.320

Source: Eviews's own processing

Selecting only models with statistically significant coefficients (prob. coef. < 0.05) Following the minimisation of informational crities, to estimate the variance of the optimum portfolio, we chose the GARCH Model (1.1) with the distribution of T-Student errors for Estimation of the optimal portfolio variance.

Therefore, to estimate the variance of the equal weight portfolio and the portfolio optimized by the Markowitz model we will use the GARCH Model (1.1) under the T-Student distribution of errors.

3.3.3. Equal weight Portfolio vs Optimum Portfolio

With the selected model we estimated the variance of the portfolios required for risk assessment. We estimated the value at risk on a 10-day horizon with probabilities of 95% and 99%. Volatility was calculated as radically from the amount of variances estimated in Eviews over the next 10 days. Table 3.22 presents Value at Risk for the two portfolios.

Table No. 7c-Value at Risk for portfolios

Value at Risk			Echiponderat	Optim
μ 1-day			0.06%	0.06%
μ 10-day			0.57%	0.57%
μ anuală			14.35%	14.35%
VaR	10-days	95%	4.03%	3.80%
VaR	10-days	99%	5.70%	5.38%
VaR	1-day	95%	1.28%	1.20%
VaR	1-day	99%	1.80%	1.70%

Source: Eviews's own processing

It can be observed that Value-at-Risk has the lower value for the portfolio optimized by the Markowitz method for each time horizon and probability considered. For example, the maximum value that can be lost by an investor who chooses the team's portfolio on a 10-day horizon and a probability of 95% is 4,032 RON (considering investment 1 RON), while the maximum loss under the same conditions is 3,801 RON if it invests in the optimal portfolio. We can see that investing in the optimized portfolio with the same expected profitability as the annual average of the equal weight portfolio, the value at risk is lower.

In conclusion, the impact of portfolio optimization has a positive impact. The value of the risk is lower for the same expected profitability. Moreover, the characteristics of the distribution of yields indicate that for the equal weight portfolio the risk is much more underestimated than in the case of the optimum portfolio.

Conclusions

This paper aimed to provide an analysis of the optimisation of the stock portfolio for the local capital market. Portfolio optimization is very important for both pension, investment or individual investor funds, as the allocation of each can bring surplus of profitability that can be reinvested or used for other useful purposes. Any rational investor pursues great profits to an assumed effort. Thus, over time, various ways have been developed to achieve the maximum efficiency of investments in the capital market. Markowitz was the first to develop a model in this regard, demonstrating how the risk could be minimized for a certain level of expected profitability. Further, researchers and practitioners attempted to identify optimization models that would capture the more nuanced risk (semvariance, VaR, CVaR, etc.).

Therefore, optimization models lead to different structures for the same expected profitability, their choice depending on investor risk aversion. For Riscophobes and for the neutral at risk, the Markowitz model and the Sortino model minimizing the semivariance are recommended, and for those who have high risk appetite, the C-VaR model that minimizes extreme losses is recommended. Where there is a possibility of trading the risk-free asset, it is recommitted to its inclusion in the portfolio since the local market yields are very high. In the last part of this work we analyzed the impact of optimizing on VaR, comparing the equimerised portfolio with Portofolile optimized by Markowitz for the same level of profitability expected. The analysis of these portfolios and the results of the VaR showed that the optimization by Markowitz reduces the value at risk and tends to underestimate less that value than in the case of the equipment portfolio.

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