A LITERATURE REVIEW ON THE GOAL PROGRAMING METHODS USED FOR PORTFOLIO OPTIMIZATION

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Abstract

Portfolio optimization as method used in finance aims to identify the potential scenarios for satisfying conflicting/competing objectives like, for example, maximizing the return/profit and reducing the risk/loss. Thus, the motivation for choosing studying and applying this method arises from the potential scenarios that the portfolio managers or the individual investors have when applying this framework. Obviously, the decision itself will belong to these decision makers, in terms of yield / risk limits, and traders will achieve the execution of this goal. The analysis per se of the polynomial branch of the goal programming will be done to indicate the achievement of the limitations which are heavily mentioned since the very beginning of the modern theories of portfolio management, limitations given by the needs of incorporating the higher order moments in the investment decision.

Keywords: portfolio selection, optimization, higher moments, polynomial goal programming, portfolio management, higher order moments

JEL Classification: C44, C61, C63, G11.

Introduction

Hereby paper aims at analysing the works in the field of portfolio goal programming with extra attention in the area of polynomial goal programming (PGP), the target being to present the researchers in this field with the current state of the literature and the research but also to present the investors with the possibility to use these methods.

The paper is structured in three sections: first we present with the historical and global context of the goal programming and with the most relevant studies at various points in time, the second part clarifies the key concepts and models used in this field and the third part aims at presenting an empirical analysis of polynomial goal programing and the conclusions.

Historical evolution of goal programming

In the financial domain, as part of the Multi Criteria Decision Analysis, goal programing is offering various portfolio management methods and methodologies both to investors and portfolio managers but also to researchers. In the context of portfolio theory, it solves problems related to either achieving results given a certain set of resources and constraints

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or determining the various resources needed to maximize results while minimizing resources by providing multiple optimal solutions/scenarios for achieving these results.

In other words, goal programming aims at solving problems related to either achieving a set of results given a set or constraints or, vice versa, determining the needed resources in order to maximise the results while minimising the resources, offering multiple optimal scenarios for achieving these results.

More specifically, as part of the portfolio theory, the works of Charnes et al (1959), Charnes and Cooper (1961) aimed, among others, at identifying the optimal portfolios for banks given various time frames.

The set of rules regarding the goal programming in general and the need to use these methods for portfolio management in particular is attributed (Tamiz et al 1998) to H.A. Simon (1957) who defines the concept of “satisficing” objectives as a decision making strategy when multiple alternatives unknown ex-ante exist and appear sequentially. Thus, Simon creates the prerequisites for using the optimization methods we are analysing by identifying the need to either simplify the decisional space into optimization problems or to identify satisfying solutions (local optimum) in taking into account a big number of characteristics (preferences) in describing the optimization problem. In other words, although goal programming was not initially intended at satisfying the decision-makers’ (investor/portfolio manager) objectives, the method gain ground and became more and more used for this particular purpose.

Romero (1991, 2004) concludes first that goal programming is among the most used methods to take a decision, with a methodology which is easy to understand and implement, further (2004) capturing the fact that the majority of research papers are using the **lexicographic** approach, the one in which weighing the preferences of decision-makers is not flexible, these preferences being mostly prioritised by importance (i.e. the first being maximising the expected return, the second being minimising the risk, the third being maximising skewness and so on), showing that this approach risks at not being suitable for empirical purposes when the lexicographic function (of prioritizing the criteria to be optimised) is not well prepared. In the same paper (2004) Romero captures other methodologies of goal programming, such as MinMAx (Chebyshev) which aims to minimize the loss of the individual optimum (i.e. the optimum yield as the only goal will be superior to that yield when taking into account the risk, skewness etc.), thus aiming to minimize the difference between the 2 sets of optimal pairs.

On the other hand, weighting the programming of objectives implies prioritizing constraints according to their importance, for example, return is always preferred to minimizing risk. The resulting optimal scenarios must take into account certain allowed deviation limits from the optimal, in other words, investors will set a certain margin of freedom in which the results may vary in reaching the optimal, i.e. a deviation of x% from an individual optimum for each constraint added to the model.

Although in the research papers from the 80s it was presented more for theoretical purposes (ie Romero, 1986), since the 2000s the method of **fuzzy** programming, aiming at dealing with uncertainty, starts to be taken into account, often this methodology being completed/complemented by simulation methods in order to identify the weights that are allocated to the optimization criteria.

Getting closer to the current period, Kalayci et al (2019) analysed a number of 175 studies related to portfolio management according to the average-variance criteria of the last 2 decades. Although the study mainly covers optimization based on the first two moments of the distribution (mean and variance), multi-objective methods are also taken into account, distinguishing between the following preference criteria: cardinality (related to the number of securities in the portfolio), trading costs, sector capitalization and so on.

Two other aspects are worth mentioning in the context of this study: first, the calculation methodologies used in the analysed studies are similar to those for multi-objective programming, and the second is that other such efforts should also be mentioned in the literature.

Omitted by the previous study are the research of Aouni et al (2014) and Colapinto et al (2017) which reveal an increased attention for goal programming in the context of portfolio theory always showing a preference for lexicographic (preemptive) and weighted methods (Archimedean). At the same time, these two studies show an increase in the absolute number of studies based on the **polynomial** method.

The concepts used in goal programming

First of all we need to remind of Henry Markowitz (1995) proposed model. Given N assets (i=1, N), an observed period T, Pi(t) representing the price of asset i at time t, the return of asset i at moment t is Ri(t) as shown in equation (1):

$$R_i(t) = \ln\left(\frac{P_i(t)}{P_i(t-1)}\right) \quad (1)$$

The expected return for this asset i is provided by equation (2):

$$E[R_i] = \frac{1}{T}\sum_{t=1}^{T} R_i(t) \quad (2)$$

The variance for asset i's returns, meaning the measure of risk is represented in equation (3). The covariance between any 2 assets (i,j) is computed in equation (4).

$$\sigma_i^2 = E[R_i(t) - E(R_i)]^2 = \frac{1}{T}\sum_{t=1}^{T}[R_i(t) - E(R_i)]^2 \quad (3)$$

$$\operatorname{cov}(R_i, R_j) = E[(R_i(t) - E(R_i))(R_j(t) - E(R_j))] = \frac{1}{T}\sum_{t=1}^{T}[(R_i(t) - E(R_i))(R_j(t) - E(R_j))] \quad (4)$$

We can define the expected return and risk of any given portfolio using equations (5) and (6), where w_i represents the relative weight of asset i in the portfolio.

$$E[R_P] = \frac{1}{T}\sum_{i=1}^{I} w_i E(R_i) \quad (5)$$

$$\sigma_P^2 = \sum_{i=1}^{I} w_i^2 \sigma_i^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{I} w_i w_j \operatorname{cov}(R_i R_j) \quad (6)$$

Following Markowitz's theory, the optimization model is given by equation (7). In this context, and investor would choose for the portfolio with the highest return at a given risk or the other way around.

$$\max_{w_i} E(R_P) = \sum_{i=1}^{I} w_i E(R_i)$$

$$\begin{cases} \sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \leq c \\ \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0, \quad i = 1, 2, \ldots, N \end{cases} \quad (7)$$

The optimization of the problem presented in equation (7) is done taking into account the hypothesis of normal distribution of returns, a hypothesis that is rejected by most researchers since Mandelbrot (1963) and Fama (1965) thus advancing the need to address the problem of asymmetry (skewness) and that of excess kurtosis in empirical distributions of assets' returns. Other assumptions are made in order to find a solution for this model, such as the lack of transaction costs and the financial efficiency of the capital markets. Subsequent empirical studies, most of them on emerging markets, reject these hypotheses from Markowitz's theory.

The simplest mathematical description of goal programming takes into account, as previously stated, the optimization (minimization / maximization) of a Z function defined as follows:

$$Opt(Z) = \sum_{i \in m} w_i d_i; \;\; w_i, d_i \;\geq 0 \quad (8)$$

A more complex definition is given by Orumie (2013) in the context of the need to formulate an efficient method of solving lexicographic programming:

$$Opt(Z) = \sum_{i \in m} w_i p_i (d_i^- + d_i^+); \;\; s.r. \;\; w_i, p_i \;\geq 0 \quad (9.1)$$

subject to:

$$\sum_j^n a_{ij} x_{ij} + d_i^- - d_i^+ = b_i; i = 1, \ldots m \quad (9.2)$$

$$x_{ij}, d_i^-, d_i^+ \geq 0; \; w_i > 0; \; (i = 1, \ldots, m; \; j = 1, \ldots n \quad (9.3)$$

d_i^- and d_i^+ repectively representing the negative and positive deviation variables, representing at the same time the quantification of the sub/over-achievement of the optimal target for a certain criterion (sub-objective) b_i representing the individual targets for each criterion taken into account in the optimization function.

Obviously, as previously described, in the case of lexicographic programming d_i, the preference parameters, are ranked $d_i < d_{i+1}$ thus giving the investor / portfolio manager the opportunity to express their preferences.

Getting back to Markowitz's model, by applying the lexicographic model we can describe the following optimization function:

$$Opt(Z) = d_1 \sum MaxE(R_p) + d_2 \sum Min\sigma_p^2; d_1 > d_2; d_1, d_2 \geq 0 \quad (9)$$

The simplified version of the MinMax model (Chebyshev) is shown below, opt' representing the local optimum, given the constraint to optimize both objectives simultaneously:

$$Opt(Z) = Min\left(optE\left(R_p\right) - opt'E\left(R_p\right)\right) + Min\left(opt\,\sigma_p^2 - opt'\sigma_p^2\right) \quad (10)$$

The representation of the weighted goal programming model is shown as:

$$Opt(Z) = \sum_{i \in m}(w_i^- {}_i d_i^- + w_i^+ {}_i d_i^+) \quad (11), \text{ s.t. } (9.2), (9.3)$$

with w_i^- and w_i^+ being the weights associated to the negative and positive deviations respectively.

Polynomial goal programming

Polynomial goal programming (PGP) is a technique that allows us, among other things, to incorporate higher order moments (skewness and kurtosis) in the selection and management of the portfolio. The ultimate goal of the PGP model is to minimize deviations between the optimum of each objective and the aggregate final objective. Among the advantages of using this model are: the existence of an optimal solution, the flexibility to incorporate investors' preferences and the relative simplicity of calculation methods that are use, and the fact that the model is general enough to include investors' preferences related to higher order moments, skewness and kurtosis.

As shortcomings of the multi-objective programming model we identify the fact that the preference parameters are randomly entered in the model, not actually taking into account the real preferences of investors. Thus the working hypotheses in which a set of natural numbers taken independently of market preferences can be considered rather restrictive, although they facilitate the design and interpretation of the model, they can significantly distort the empirical results. The solutions for this limitation came from Davies and Kat (2009) who identified and then developed the solution proposed by Lai (1991), that of using the marginal rate of substitution between objectives, and especially through the work of Proelss and Schweizer (2009) which, processing a sample from over one hundred hedge funds, empirically identified investors' preferences for the various higher order moments, further managing to transpose the parameters identified in the multi-objective programming model.

We define the spatial components of multi-objective optimization as follows[*]:

- Decisional space – having the subset of decisions with feasible or implementable solutions;
- Objectives' space – which represents the variable(s) that are actually optimised;

[*] Jones, D. (2011), "A practical weight sensitivity algorithm for goal and multiple objective programming", *European Journal of Operational Research*, pp. 238-245.

- Parameters' space – having all the possible parameters that are included in the multi-objective model;
- Weights space – the set of preferences parameters used in the goal programming.

Thus we're defining the objectives' space as being:

$$Mean = M(x) = X^T \overline{M}$$
$$Variance = V(x) = X^T VX$$
$$Skewness = S(x) = E\left(X^T\left(M - \overline{M}\right)\right)^3$$
$$Kurtosis = K(x) = E\left(X^T\left(M - \overline{M}\right)\right)^4$$

where: M is the returns' distribution, $\overline{M}$ is their median, X^T=(x_1, x_2, ...,x_n) is the transposed vector o the assets' weights in the portfolio, V, S and K are the variance-covariance, skewness-coskewness and kurtosis-cokurtosis matrices of M.

To combine these parameters that form the multiple objective and maximize the expected return, including skewness while minimizing the even order moments of the distribution, we divide the optimization problem into two steps, in P1 calculating the individual optimum for each of the 4 moments:

$$P_1\left\{\begin{array}{l} Max(Mean = M(x) = X^T \overline{M}) \\ Min(Variance = V(x) = X^T VX) \\ Max\left(Skewness = S(x) = E\left(X^T\left(M - \overline{M}\right)\right)^3\right) \\ Min\left(Kurtosis = K(x) = E\left(X^T\left(M - \overline{M}\right)\right)^4\right) \\ \text{subject to: } X^T I = 1 \\ X \geq 0 \end{array}\right.$$

To combine these objectives into a single objective function, according to the PGP methodology we need d1, d2, d3 and d4 parameters, the variables that quantify the deviations of the mean, variance, skewness and kurtosis from the optimal values M *, V *, S * and K * respectively. To obtain the optimal score, the P1 model is divided into four sub-problems that are solved individually.

After calculating the optimal level of each moment, we move on to step 2 (P2) having the next optimization with the corresponding restrictions[†]:

$$Min\left(Z = (1 + d_1)^{\lambda 1} + (1 + d_2)^{\lambda 2} + (1 + d_3)^{\lambda 3} + (1 + d_4)^{\lambda 4}\right)$$

$$P_2\left\{\begin{array}{l} \text{SR: } X^T \overline{M} + d_1 = M^* \\ X^T VX - d_2 = V^* \\ E\left(X^T\left(M - \overline{M}\right)\right)^3 + d_3 = S^* \\ E\left(X^T\left(M - \overline{M}\right)\right)^4 - d_4 = K^* \\ \text{subject to: } X^T I = 1 \\ X \geq 0 \\ d_i \geq 0; i = 1, \dots, 4 \end{array}\right.$$

† Davies, R.; Kat, H.; Lu, S., (2003), „Fund of hedge funds portfolio selection: A multiple-objective approach", Journal of Derivatives & Hedge Funds, nr. 15, pg 91–115.

To illustrate multi-objective programming, we present here some empirical results, presented in a previous study[‡]:

Table 1: Moments preferences (lambda)

	M	V	S	K
Scăzut	1,5	2	1,2	1,1
Mediu	3,5	5	2,5	1,2
Ridicat	7	10	5	1,4

Source: own computations

Thus, from table no. 1 we can identify the preference parameters (lambda) of the investors proposed for BSE for each of the moments of distribution (MVSK). Table no. 2 reveals the results of the application of these parameters and the results of the 4 moments of distribution (MVSK) for a portfolio of 20 securities (selected in compliance with the criteria of liquidity selection, stock market capitalization and diversification by activity sectors).

Table 2: Optimal results for the Moments* of the returns

M	Medium	High	High	High			
V	Medium	High	Medium		High		
S	Medium	Low	Low			High	
K	Medium	Low	Low				High
SNP	28%	25%	6%		13%		
TGN		6%			8%		
BIO	4%	10%				91%	
SIF5			9%				
TBM	3%						
ALT	18%	2%	39%			3%	30%
ALU		5%					
RRC	12%	6%	22%		16%		32%
SCD	29%	21%	11%		19%	6%	
EBS	5%	9%	12%				18%
CMP	1%	17%	2%	100%	22%		
M	0,0010	0,0013	0,0011	0,0021	0,0010	0,0014	0,0009
V	0,0155	0,0150	0,0191	0,0297	0,0127	0,0216	0,0191
S	0,3627	0,0433	0,4553	0,6702	0,1961	1,0700	0,2305
K	3,3372	5,7119	2,6767	6,4203	4,5750	9,5923	1,7327

As one of the objectives of the optimization system improves, at least one of the other objectives deteriorates as a degree of preference for investors. TLV, BRD, TEL, BRK, DAFR, SIF3 and SIF4 securities are left out from the portfolios. The conclusion is that they have mediocre values for all 4 moments of the distribution of return. In fact, these securities have the highest values for sensitivity to market fluctuations (Beta > = 1.2). Although

[‡] Bahna M. and Cepoi C.O., "Optimizarea selecției și gestiunii portofoliului folosind momentele de ordin superior", RSF no 1, 2016

preferred for its superior return, having a total allocation in the portfolio that strictly optimizes the return, CMP fails to shine when moments of order 3 and 4 are taken into account.

At the opposite end SNP, ALT, RRC, SCD and EBS assets are the most selected by investors according to the model. RRC and SCD are selected in a high proportion. These securities, strongly linked to investor preferences, are inelastic in terms of return fluctuations compared the market fluctuations, with a beta of up to 0.51.

We present below only parts of the conclusions from our previous study, related to the application of the PGP methodology in optimizing the portfolio management:

- The issues related to the computational part remain among the most difficult in applying the method;
- Successful identification/computation of optimal weights related to preferences for higher order moments can be made;
- Identified preference parameters can be reused when higher order moments are taken into account, at least for the BSE case.

We add here some conclusions from recent studies, Livingston (2009) makes efforts to justify the use of the PGP method, showing the advantages given by the flexibility of choosing this method and the diversification of options available to the investors. Khan (2020) identifies a better frontier of efficiency and the possibility of substantiating the investment decision. Chen (2020) concludes as positive aspects of the methodology: stabilization of cumulative returns and a better response to market downturns. On the other hand, he points out that the calculation time, compared to similar methodologies, is longer, but reasonable for the results provided. Cizauskas and Haslifah (2019) emphasize in their study that the use of mean, variance and skewness, as parameters in the model, outperform other management methods using higher order moments. Gupta et al (2019) conclude on the advantages related to flexibility in decisions made by investors by using the parameters of mean, variance, skewness and entropy, showing instead that the calculation time for applying the model is longer. Other recent studies show that taking higher order moments into account while using the PGP method reduces exposure to entropy and uncertainty, respectively.

We also conclude that the shortcomings of the multi-objective programming model, often related to the calculation time or the accurate identification of the real preferences of investors, are offset by the advantages of using the goal programming methodology in general and PGP in particular. The advantages given by the flexibility of the model are also the facilitation of the investment decision and especially of the portfolio selection and management taking into account self-preference criteria. In addition, the possibility of incorporating the higher order moments of the returns' distribution allows for a more informed decision to be taken with respect to future earnings, the portfolio being less exposed to risks of uncertainty, for example. We cannot ignore the computing power needed to solve these proposed models by using the objective optimization methodology. What we can say for sure is that the methodology itself is one that aims to solve complex problems, in quadratic space in order to help the investors to treat them in linear space. But no matter how much the computational speed of these models is optimized, the research must identify more and more complex models, requiring new optimizations in the computational part itself.

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